Tema 13. Áreas y volúmenes

1. Unidades de volumen:

- Volumen es la cantidad de espacio que ocupa un cuerpo.

★ La unidad fundamental es el $\boxed{m^3}$

Es el volumen de un cubo que tiene 1 m de arista.

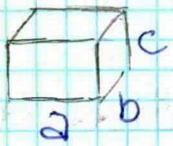
CAPACIDAD Y VOLUMEN

$\boxed{1L = 1\ dm^3}$

$1m^3 \rightarrow$	$1\ dm^3 \rightarrow$	$1\ cm^3$
1 KL	1L	1 mL

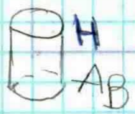
2. Área y volumen del ORTOEDRO, PRISMA y CILINDRO.

★ Ortoedro: $\boxed{V = abc}$



★ Prisma: $\boxed{V = A_B \cdot H}$

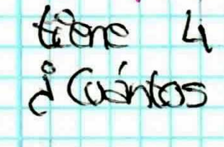
★ Cilindro: $\boxed{V = A_B \cdot H}$



Ejemplo 1: Una piscina tiene 20 m de largo, 10 de ancho y 2 de profundidad. ¿Cuál es su volumen?

$V = 20 \cdot 10 \cdot 2 = \boxed{400}\ m^3 = 400.000\ L$

Ejemplo 2: Un depósito de gasoil tiene forma cilíndrica tiene 4 metros de largo y 1 m de diámetro. ¿Cuántos litros contiene?



$V = A_B \cdot H = 0'785 \cdot 4 = \boxed{3'14}$

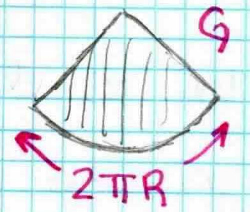
$A_B = \pi \cdot R^2 = \pi \cdot 0'5^2 = 3'14 \cdot 0'25 = \boxed{0'785}\ m^2$

3. Área y volumen de PIRÁMIDE, CONO y ESFERA.

- Pirámide: $V = \frac{1}{3} A_B \cdot H$

- Cono: $V = \frac{1}{3} A_B \cdot H$

Superficie:

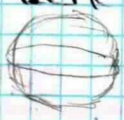


$A_L = \frac{2\pi R \cdot G}{2} = \boxed{\pi R G}$

- : $A = 4 \cdot \pi \cdot R^2$

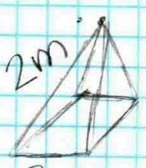
$V = \frac{4}{3} \cdot \pi \cdot R^3$

Ejemplo 1: Un balón tiene de radio 10 cm. ¿Qué volumen tiene?



$V = \frac{4}{3} \cdot \pi \cdot R^3 = \frac{4}{3} \cdot 3'14 \cdot 10^3 = \frac{4}{3} \cdot 3'14 \cdot 1000 = \boxed{4186}\ cm^3$

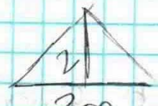
Ejemplo 2: Una tienda de campaña tiene forma de pirámide cuadrangular. Tiene 3 m de arista de la base y 2m de altura. Calcula su superficie y su volumen.



$\boxed{A_t = A_B + A_L}$

$A_B = 3^2 = \boxed{9}\ m^2$

$A_L =$



$\rightarrow h^2 = 2^2 + 1'5^2$

$h^2 = 4 = 2'25$

$h^2 = 6'25 ; h = \sqrt{6'25} = \boxed{2'5}$

$V = \frac{1}{3} \cdot 9 \cdot 2$

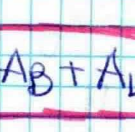
$\downarrow$

$\boxed{6}\ m^3$

$A_L = 4 \cdot \frac{3 \cdot 2'5}{2} = 2 \cdot 3 \cdot 2'5 = \boxed{15}\ m^2$

$A_L = 9 + 15 = \boxed{24}\ m^2$

Ejemplo 3: Hallar el volumen y la superficie de un cono de $R = 2$ m y $H = 5$m.



$V = \frac{1}{3} \cdot A_B \cdot H = \frac{1}{3} \cdot 12'56 \cdot 5 = \boxed{20'94}$

$A_T = A_B + A_L$

$A_L = \pi \cdot R \cdot G = 3'14 \cdot 2 \cdot 5'4$

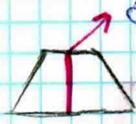
$\downarrow$

$A_B = \pi \cdot R^2 = 3'14 \cdot 2^2 = 12'56\ m^2$

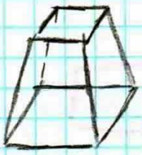
4. troncos de pirámide y de cono.

Tronco de pirámide:

Área = $A = A_{B_1} + A_{B_2} + A_L$

$A_L =$  apotema del tronco.
Trapecios isósceles.

Ejemplo: Sea el tronco de pirámide cuadrangular de aristas: 14 m (B) 4 m (b) y 12 (h)



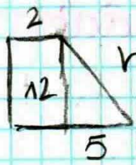
$$A = A_{B_1} + A_{B_2} + A_L$$

$$A_{B_1} = \ell^2 = 14^2 = 196 \ m^2$$

$$A_{B_2} = \ell^2 = 4^2 = 16 \ m^2$$

$$A_L = 4 \cdot \frac{(B+b)}{2} \cdot h$$

$$4 \cdot \frac{(14+4)}{2} \cdot 13 = 468 \ m^2$$



$h^2 = 5^2 + 12^2$;

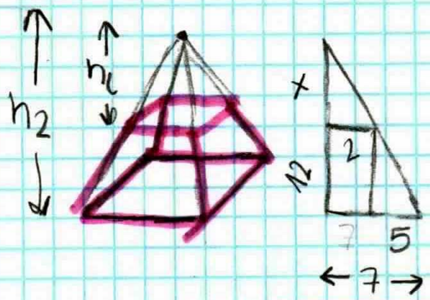
$h^2 = 25 + 144$;

$h^2 = 169$; $h = \sqrt{169} = \boxed{13}$

$$A_t = 196 + 16 + 468 = \boxed{680} \ m^2$$

$\boxed{V = V_{p_1} + V_{p_2}}$ La razón de semejanza me da:

$$\begin{array}{ccc} h_2 & & H \\ & \times & \\ a_2 & & a_1 - a_2 \end{array}$$



Ejemplo: Aristas son 14 m y 4 m altura de 12 m

$$\begin{array}{ccc} X & & 12 \\ & \times & \\ 2 & & 5 \end{array}$$

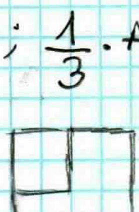
$$X = \frac{12 \cdot 2}{5} = \frac{24}{5} = 4'8$$

$h_2 = 4'8$

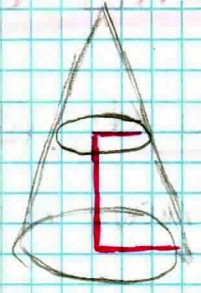
$h_1 = 12 + 4'8 = 16'8$

$\boxed{V = V_{p_1} - V_{p_2}}$; $V = \frac{1}{3} \cdot A_B \cdot H$; $\frac{1}{3} \cdot A_{B_1} \cdot h_1 - \frac{1}{3} \cdot A_{B_2} \cdot h_2$

$$\frac{1}{3} \cdot 14^2 \cdot 16'8 - \frac{1}{3} \cdot 4^2 \cdot 4'8$$



Tronco de cono:

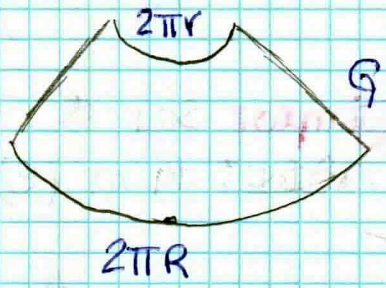


Superficie $A = A_{B_1} + A_{B_2} + A_L$

$\boxed{A_{B_1} = \pi \cdot R^2}$ $\boxed{A_{B_2} = \pi r^2}$

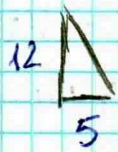
$A_L = (2\pi R + 2\pi r) \cdot \quad =$

$\dfrac{\cancel{2}\pi (R+r) \cdot G}{\cancel{2}} = \boxed{\pi (R+r) \cdot G}$



Ejercicio: Superficie del tronco de cono= $R = 7$ $r = 2$ $H = 12$

$A = A_{B_1} + A_{B_2}$; $A_{B_1} = \pi \cdot 7^2 = 3'14 \cdot 49 = \boxed{153'9}$



$A_{B_2} = \pi \cdot 2^2 = 3'14 \cdot 4 = \boxed{12'56}$

$A_L = \pi (7+2) \cdot G$; $\pi \cdot 9 \cdot 13 = \boxed{367'56}$

$G^2 = 5^2 + 12^2$

$G^2 = 25 + 144$

$G^2 = 169$; $G = \sqrt{169} = 13$

$A = 153'9 + 12'56 + 367'56 = \boxed{534'02}\ m^2$

Volumen: $V = V_{C_1} - V_{C_2}$

$\dfrac{1}{3} \cdot \pi \cdot 7^2 \cdot 16'8 - \dfrac{1}{3} \pi \cdot 2^2 \cdot 4'8$ $V_C = \dfrac{1}{3} \cdot A_B \cdot h$

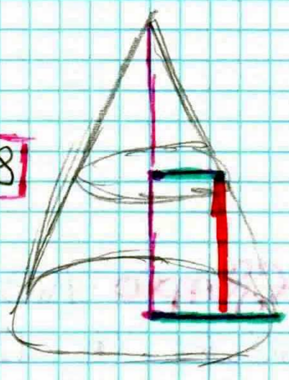
$\downarrow$

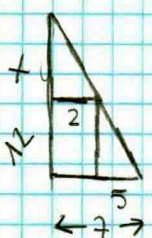
841'954

$\dfrac{x}{r} \times \dfrac{H}{R-r}$

$\dfrac{x}{2} \times \dfrac{12}{5}$

$x = \dfrac{2 \cdot 12}{5} = \dfrac{24}{5} = \boxed{4'8}$





1.259) Transforma mentalmente en m^3

B

a) $25\ dam^3 = 25000\ m^3$ b) $2560\ dm^3 = 2'56\ m^3$
c) $45\ Km^3 = 45000000000\ m^3$ d) $0'02\ hm^3 = 20000\ m^3$
e) $32000\ cm^3 = 0'032\ m^3$ f) $575000\ mm^3 = 0'000575\ m^3$

2.259) Expresa en L las siguientes cantidades:

B

a) $5\ m^3 = 5000\ L$ b) $0'008\ hm^3 = 8000\ L$

c) $250\ dm^3 = 250\ L$ d) $12000 = 12\ L$ e) $10\ Km = 1 \cdot 10^{12}$

f) $25000\ mm^3 = 0'25\ L$

8.261) Calcula el área de un tetraedro de 6 m de arista.

$A = 2(a \cdot b + a \cdot c + b \cdot c)$
$A = 2(10 \cdot 3 + 10 \cdot 5 + 3 \cdot 5)$
$A = 2(30 + 50 + 15)$
$A = 2(95)$
$A = \boxed{190}\ m^3$

$V = A \cdot b \cdot c$
$V = 10 \cdot 3 \cdot 5$
$V = \boxed{150}\ m^3$

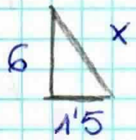
B

9.261) Área y volumen de prisma rectangular

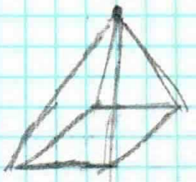
$A_t = 2A_B + A_L$
$A_B = 3 \cdot 3 = 9\ m^2 \cdot 2 = 18\ m^2$
$A_L = 8 \cdot 3 = 24\ m^2$
$A_t = 18 + 24 = \boxed{42}\ m^2$

$V = A_B \cdot H$
$V = 9 \cdot 8 = \boxed{72}\ cm^2$

13.263) Haz el dibujo y halla el área y el volumen de una pirámide cuadrangular cuya base = 3 m de arista y H = 6 m. Aproxima el resultado a dos decimales.



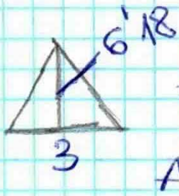
$A_t = A_B + A_L$



$A_L = 3^2 = 9\ m^2$ | $A_L = x^2 = 1'5^2 + 6^2$

$x^2 = 2'25 + 36$

$x^2 = 38'25$; $x = \sqrt{38'25} = \boxed{6'18}$



$A_L = \frac{b \cdot a}{2}$

$A_L = \frac{3 \cdot 6'18}{2} = \frac{18'54}{2} = \boxed{9'27} \cdot 4 = 37'08$

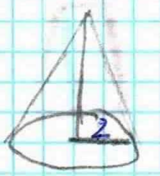
$A_B + A_L = 9 + 37'08$

↓

$46'08\ m^2$

$V = \frac{1}{3} A_B \cdot H$; $V = \frac{1}{3} 9 \cdot 6 = \boxed{18}\ m^3$

14.263) Haz el dibujo y halla el área y el volumen de un cono recto en el que el radio de la base mide 2 m y H = 8 m. Aproxima a dos decimales.



$A_t = A_B + A_L$

$A_B = \pi \cdot R^2$; $3'14 \cdot 2^2$;

$3'14 \cdot 4$;

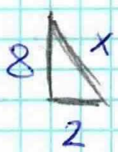
↓

$\boxed{12'56}$

$A_L = \pi \cdot R \cdot G$;

$3'14 \cdot 2 \cdot 8'24 =$

$\boxed{51'77}$



$x^2 = 8^2 + 2^2$

$x^2 = 64 + 4$

$x^2 = 68$; $\sqrt{68} = \boxed{8'24}$

$A_t = 12'56 + 51'77 = \boxed{64'33}$

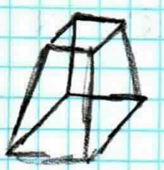
$V = \frac{1}{3} A_B \cdot H$; $\frac{1}{3} 12'56 \cdot 8 = \boxed{33'5}$

15.263) Haz el dibujo y calcula el área y el volumen de una esfera de 6 cm de radio. Aproxima a dos decimales. $A = 4\pi R^2$; $4 \cdot 3'14 \, 6^2$; $4 \cdot 3'14 \cdot 36 = \boxed{452'3}\ cm^2$

$V = \frac{4}{3} \pi R^3$; $\frac{4}{3} \cdot \pi \cdot 6^3$; $\frac{4}{3} \cdot \pi \cdot 216 = \boxed{904'7}\ cm^2$

17.265 Dibujo, área y volumen de tronco de pirámide

$B = 14 \quad b = 4 \quad H = 12$



$A_t = A_{B_1} + A_{B_2} + A_L$

$A_{B_1} = l^2 = 14^2 = 196 \ m^2$

$A_{B_2} = l^2 = 4^2 = 16 \ m^2$

$A_L = \dfrac{(B+b) \cdot h}{2} = \dfrac{(14+4) \cdot 12}{2} = \dfrac{216}{2} = 108 \ m^2$

$108 \cdot 4 = 432$

$A_t = 196 + 16 + 432 = \boxed{644}$

$V = \dfrac{1}{3} (A_{B_1} + A_{B_2} + \sqrt{A_{B_1} \cdot A_{B_2}}) \cdot H$

$\dfrac{1}{3} (196 + 16 + \sqrt{196 \cdot 16}) \cdot 12 = \boxed{1744} \ m^3$

18.265 Dibujo, área y volumen tronco de cono

$B = 10 \quad r = 4 \quad h = 15$

$A_t = A_{B_1} + A_{B_2} + A_L$

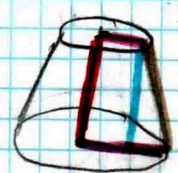
$A_{B_1} = \pi R^2 = \pi \cdot 10^2 = \pi \cdot 100 = 314'15$

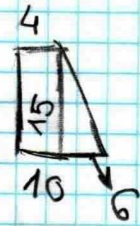
$A_{B_2} = \pi r^2 = \pi \cdot 4^2 = \pi \cdot 16 = 50'3$

$A_L = \pi \cdot (R+r) \cdot G$

$\pi \cdot (10+4) \cdot 16'15$

$\pi \cdot 14 \cdot 16'15 = \boxed{710'3} \ m^2$





$G^2 = 15^2 + 6^2$

$G^2 = 225 + 36$

$G^2 = 261;$

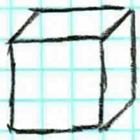
$G = \sqrt{261} \cong 16'15$

$A_t = 314'15 \circ 50'3 \cdot 710'3 = \boxed{1074'8}$

$V = \dfrac{1}{3} (A_{B_1} + A_{B_2} + \sqrt{A_{B_1} \cdot A_{B_2}}) H$

$\dfrac{1}{3} \cdot (314'15 + 50'3 + \sqrt{314'15 \cdot 50'3}) \cdot 15 = 6280 \ m^3$

2.F) Calculate the diagonal, lateral area, surface area of a cube an edge of 5 cm.



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Diagonal = $D^2 = a^2 + b + c^2$

$D^2 = 5^2 + 5^2 + 5^2$

$D^2 = 25 + 25 + 25$

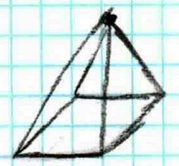
$D^2 = 75 ; D = \sqrt{75} = \boxed{8'6}$

$La = e^2$

$5^2 = 25$

$S = e \cdot 6 = 25 \cdot 6 = \boxed{150}$

7.F) Calculate the lateral area, surface area of a square pyramid whose base edge is 10 cm and its height is 12 cm.



lateral area $= \dfrac{b \cdot a}{2} ; \dfrac{10 \cdot 12}{2} = \dfrac{120}{2} = \boxed{60}\ cm^2$

Surface area $= 60 \cdot 4 = \boxed{240}$

$A_B = e^2 ; 10^2 = \boxed{100}$

5

$A_t = A_B + A_L = 240 + 100 = \boxed{340}$

8.F) Calculate the lateral area, surface area of a hexagonal pyramid with a base edge of 16 cm and a side edge of 28 cm.

$A_L = \dfrac{b \cdot a}{2} ; \dfrac{16 \cdot 28}{2} = \dfrac{448}{2} = 224\ cm^2$

$S = 224 \cdot 6 = 1344\ cm^2 \quad A_B = \dfrac{p \cdot a}{2} ; (16 \cdot 6) \cdot$